Math 2130 - Homework # 5

Double Integrals

Part 1 - Integrals over rectangles

1. Evaluate the following integrals:

   (a) $\displaystyle\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy$

   (b) $\displaystyle\int_0^2 \int_0^{\pi/2} x \sin(y)\, dy\, dx$

Part 2 - Integrals over general regions

2. Calculate $\displaystyle\iint_D (x^2 + 3y^2)\, dA$ where D is is the region in the xy-plane bounded between $x = 0$, $y = 1$, and $y = x$.

3. Calculate $\displaystyle\iint_D x\, dA$ where D is is the region in the xy-plane bounded between $x = 0$, $y = x$, and $x + y = 2$.

4. Calculate $\displaystyle\iint_D 5\, dA$ where D is is the region in the xy-plane bounded between $y = x^2$ and $y = 2 - x^2$.

5. Calculate $\displaystyle\iint_D (x^2 + y^2)\, dA$ where D is is the region in the xy-plane bounded between $x = y^2$ and $x = 9$.

Part 3 - Integrals using polar coordinates

6. Draw a picture of the region D in the xy-plane that satisfies $1 \leq r \leq 2$ and $\pi/2 \leq \theta \leq 3\pi/2$.

7. Find the volume of the solid that lies beneath the paraboloid $z = 9 - x^2 - y^2$ and above the xy-plane.

8. Find the volume of the solid that lies below the $z = 9$ plane and above the paraboloid $z = x^2 + y^2$.

9. Find the volume of the solid that lies between the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

10. Calculate
$$\iint_D e^{-x^2-y^2}\, dA$$
where R consists of all (x, y) satisfying $x^2 + y^2 \leq 9$ and $y \leq 0$.
